Gang Reduction and Youth Development

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Introduction

Gang Reduction and Youth Development (GRYD)

Questionnaire: 56 questions

- (1-5) “I get very angry and ‘lose my temper’.”
- (1-5) “It is okay to beat people up if they hit me first.”
- (T/F) “The police treat people fairly.”

Goals

To investigate the changes in time and internal spatial relationships in our dataset

- Dynamical system
- Dimensionality reduction

www.lagryd.org/summer-night-lights

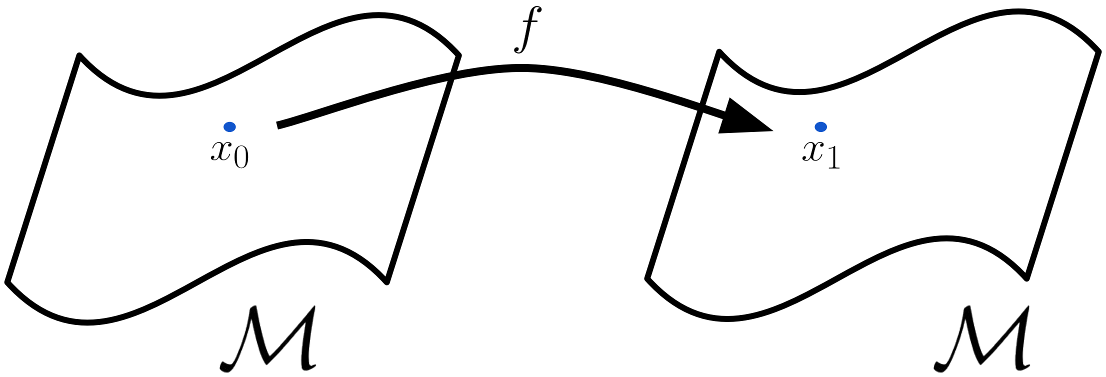
Dynamical System

Points on manifold :

$$x_t \in \mathcal{M}$$

Dynamics of the system:

$$x_{t+1} = f(x_t)$$



Dynamical System and Koopman Operator

Look at the space of vector-valued functions: $\{g : \mathcal{M} \to \mathbb{R}^m\}$

$$x \xrightarrow{\quad g \quad} g(x) = \begin{bmatrix} 5 \\ 2 \\ 4 \\ \vdots \\ 0 \end{bmatrix}$$

Participant of GRYD

Response vector of the participant

Koopman Operator Theory:

$$\mathcal{K}(g(x)) = g(f(x))$$

$\mathcal{K}$ is a linear, infinite-dimensional operator

Dynamical System and Koopman Operator

Look at the space of vector-valued functions: $\{g : \mathcal{M} \to \mathbb{R}^m\}$

$$x \xrightarrow{\quad g \quad} g(x) = \begin{bmatrix} 1 \\ 0.25 \\ 0.75 \\ \vdots \\ 0 \end{bmatrix}$$

Participant of GRYD

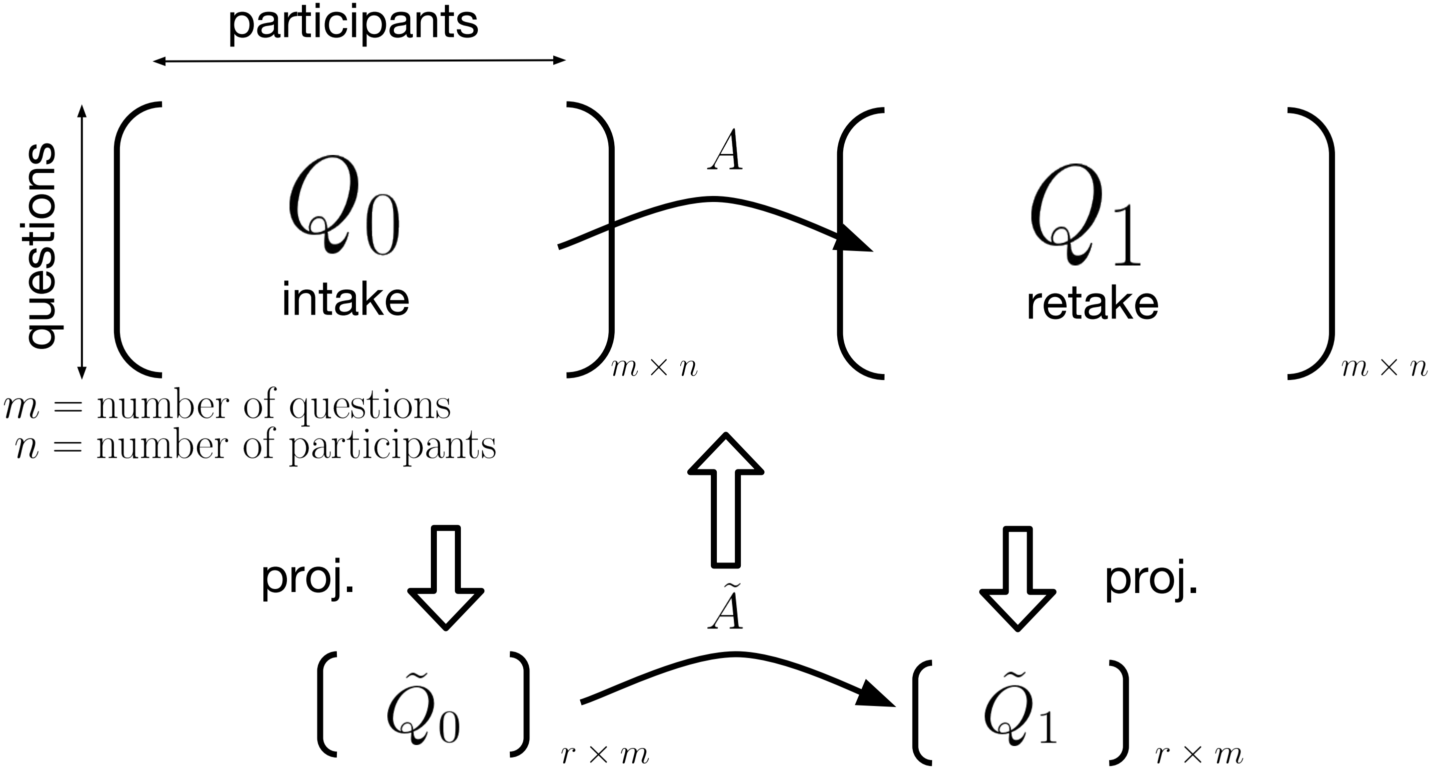
Response vector of the participant

Koopman Operator Theory:

$$\mathcal{K}(g(x)) = g(f(x))$$

$\mathcal{K}$ is a linear, infinite-dimensional operator

Dynamic Mode Decomposition (DMD)



Problem:

$$A \cdot Q_0 = Q_1$$

Least Squares Solution:

$$A = Q_1 \cdot Q_0^+$$

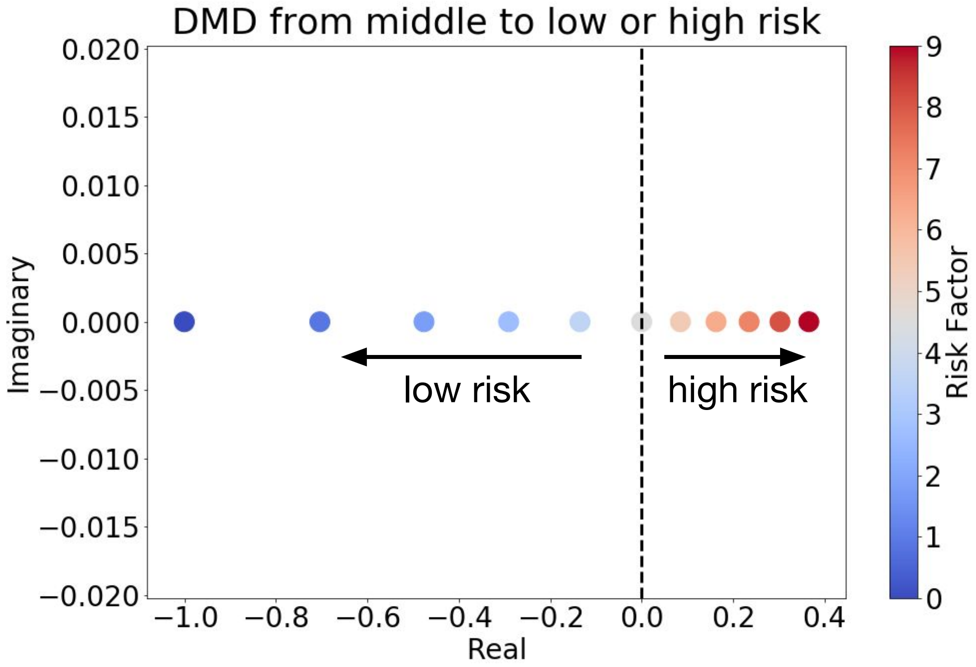
DMD Algorithm

Input: initial matrix Q_0 , final matrix Q_1 , rank r

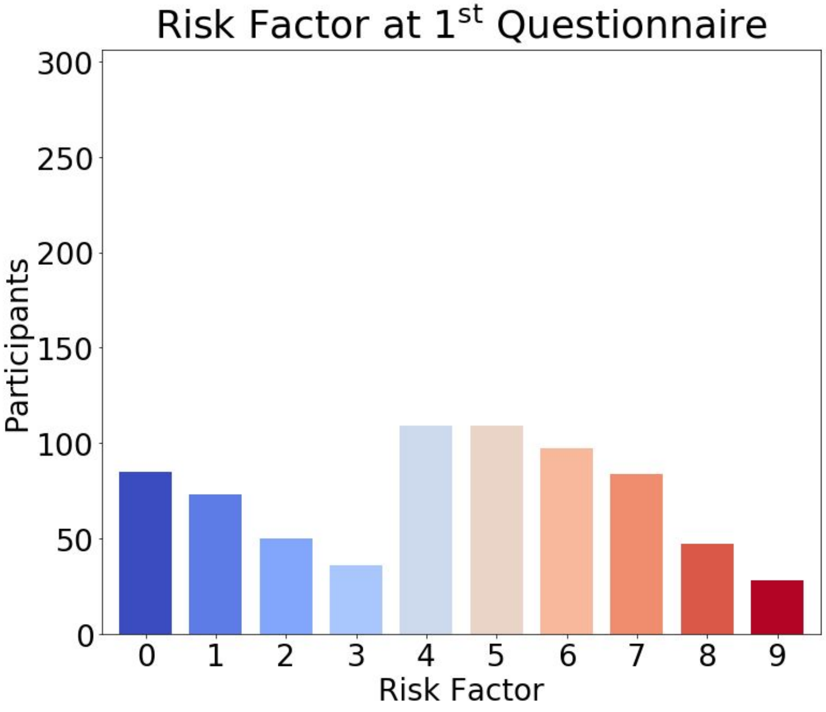
1. Compute rank-r SVD of $Q_0 = U\Sigma V^\top$;
2. Define $\tilde{A} = U^\top Y V \Sigma^{-1}$;
3. Compute the eigenvalues and eigenvectors of $\tilde{A}$: $\{\lambda_i\}_{i=1}^r$ and $\{w_i\}_{i=1}^r$
4. The eigenvalues and eigenvectors of A are given by $\{\lambda_i\}_{i=1}^r$ and $\{\phi_i = YV\Sigma^{-1}w_i\}_{i=1}^r$

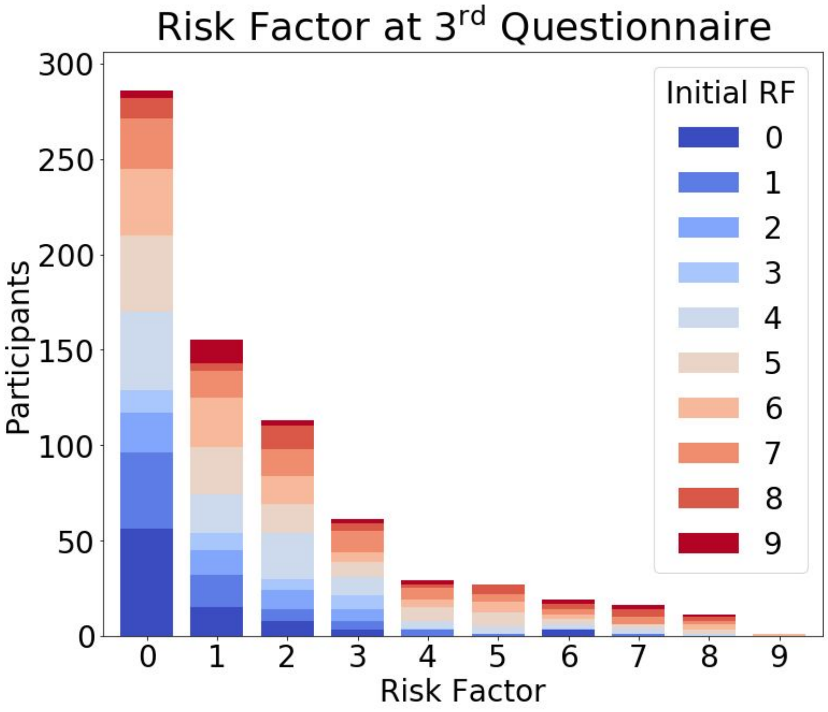
Output:
- DMD eigenvalues
  - real part: growth/decay
  - imaginary part: frequency
- eigenvectors
  - select by largest eigenvalue

DMD on Constructed Dataset

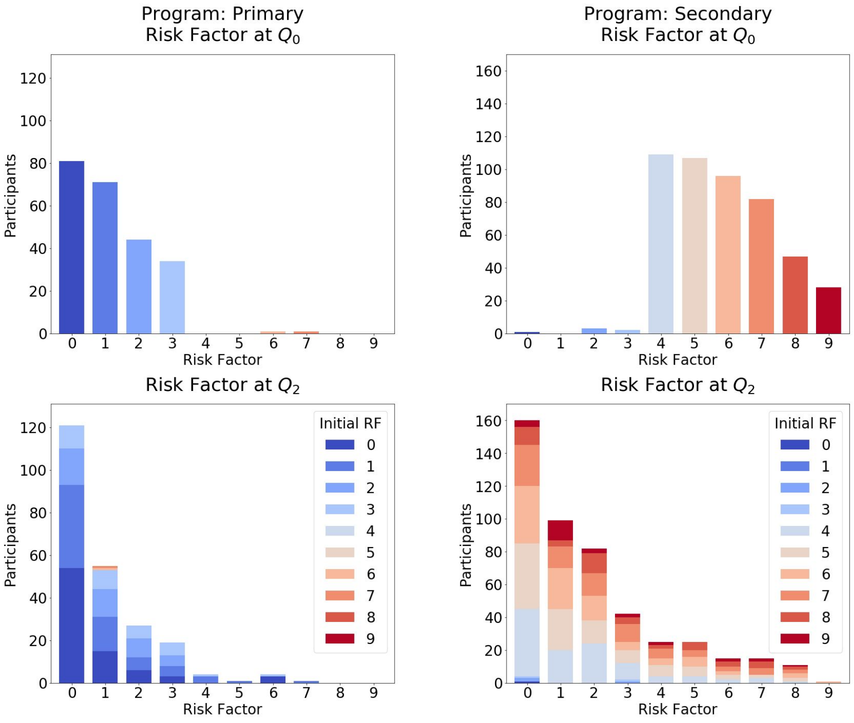


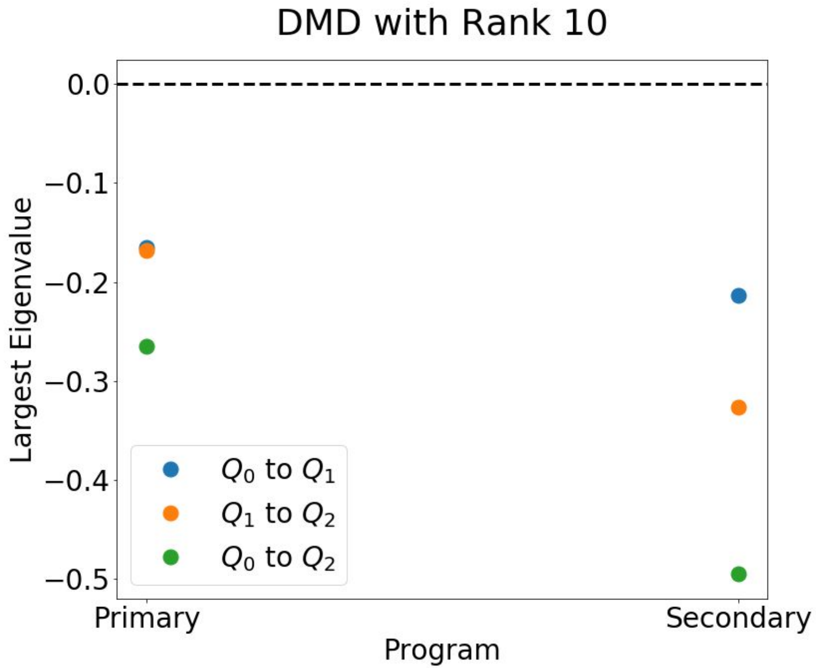
GRYD Risk Factors



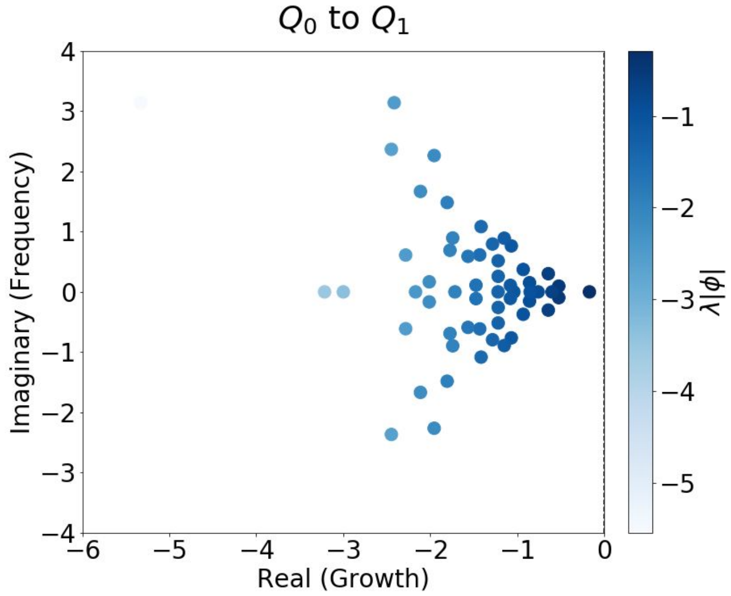


DMD by Program

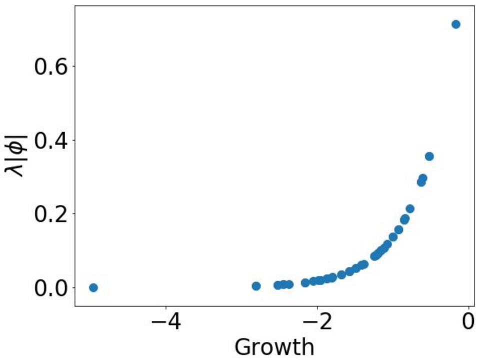


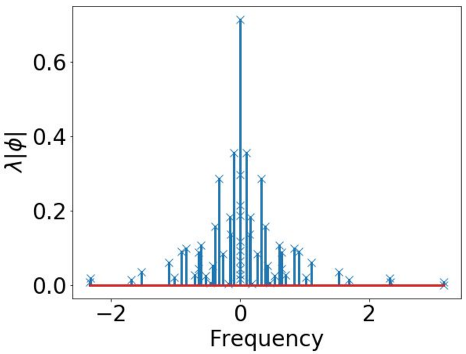


DMD Results on GRYD

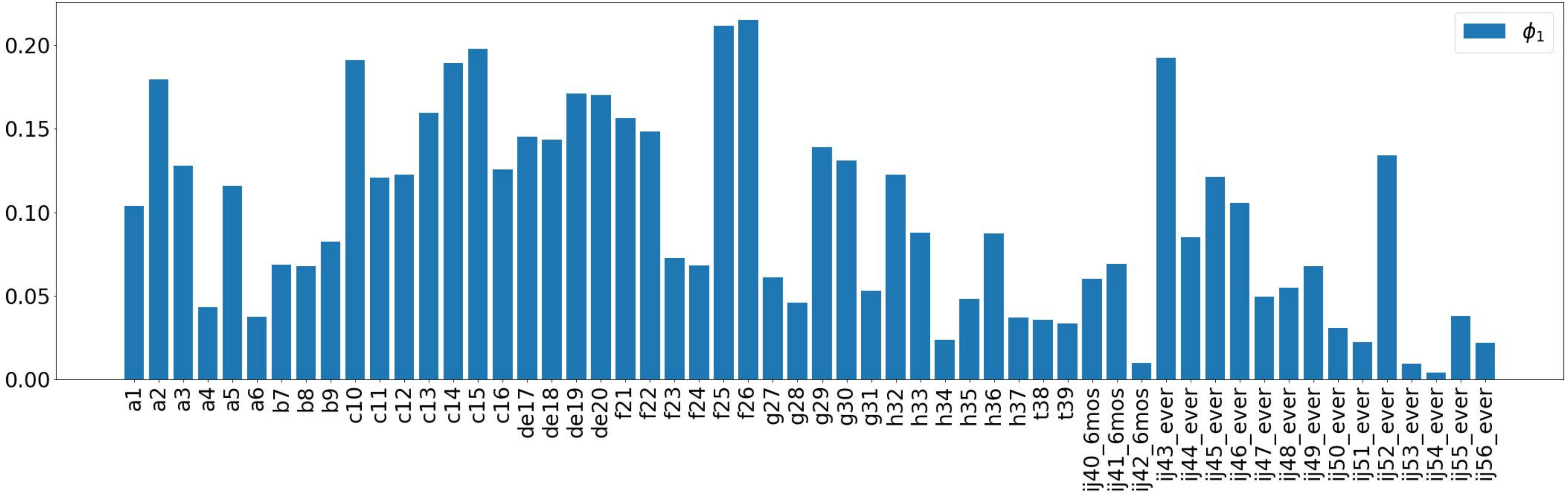


- largest log eigenvalue -0.168 < 0
- dominated by largest eigenvalue
- no apparent frequency



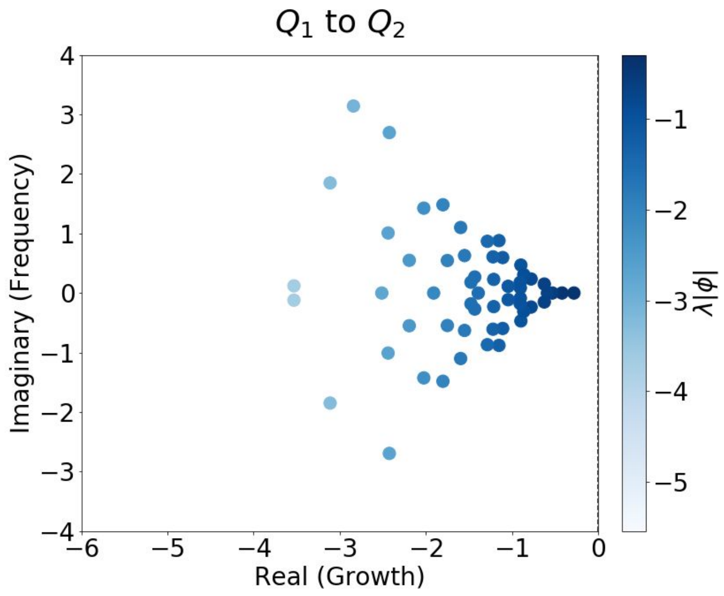


DMD Results on GRYD

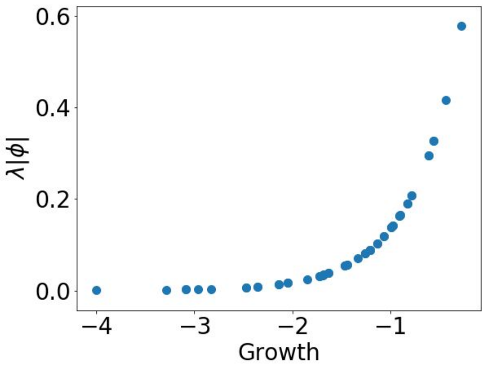


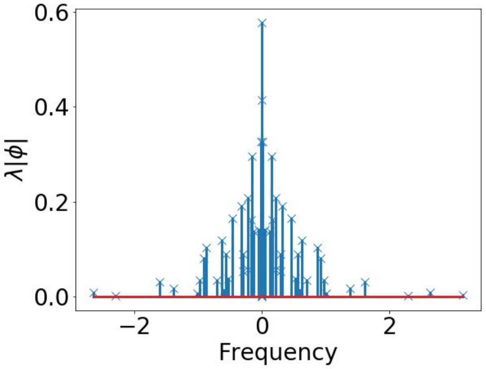
Question 25: “It is okay to beat people up if they hit me first.”
Question 26: “It is okay to beat people up if I do it to stand up for myself.”

DMD Results on GRYD

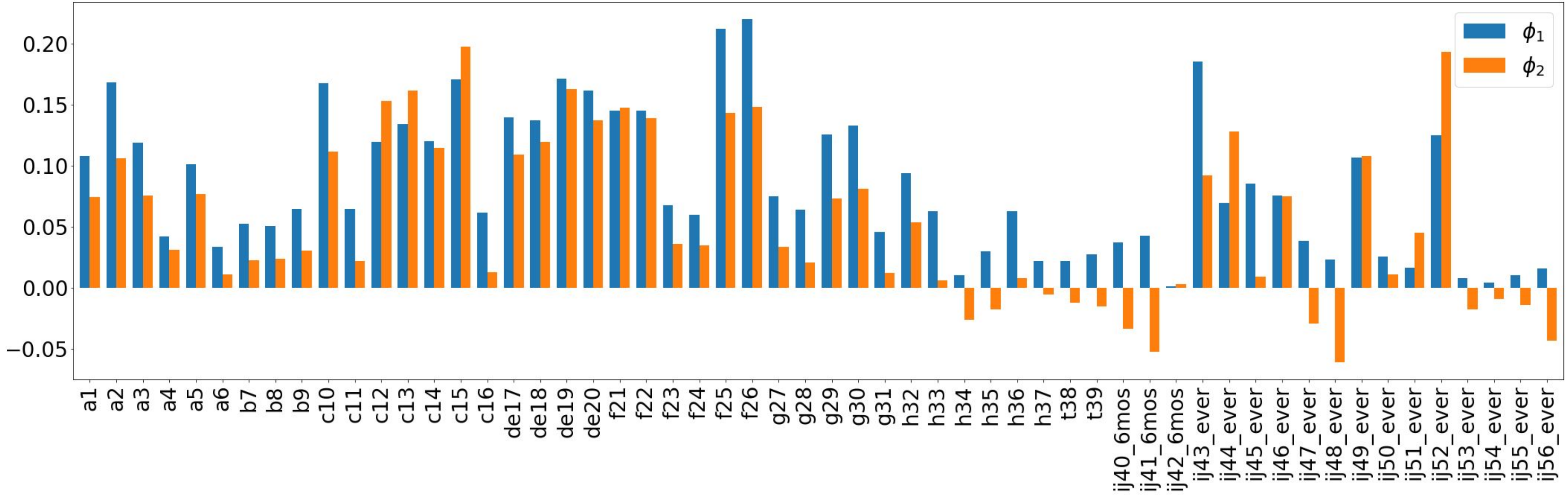


- largest log eigenvalue -0.274 < 0
  - lower than Q_0 to Q_1
- dominated by largest few eigenvalues
- no apparent frequency





DMD Results on GRYD



Question 15: “Did you start hanging out with a new group of friends?”

Question 52: “Hit someone with the idea of hurting him/her?

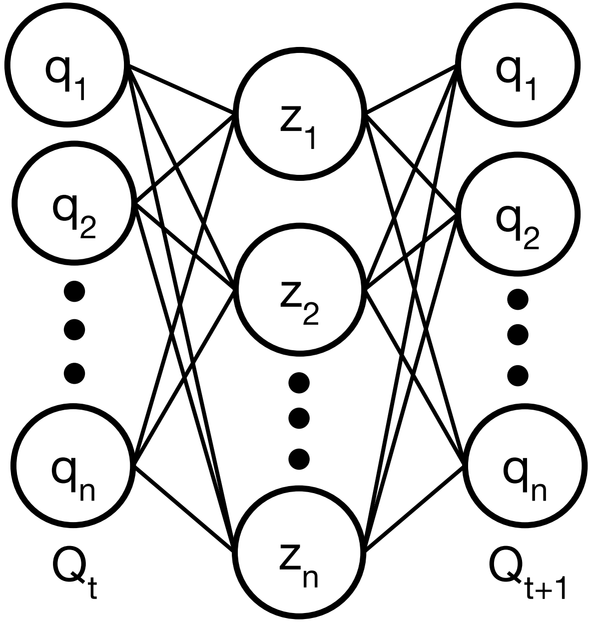
Prediction

Dynamic Mode Decomposition

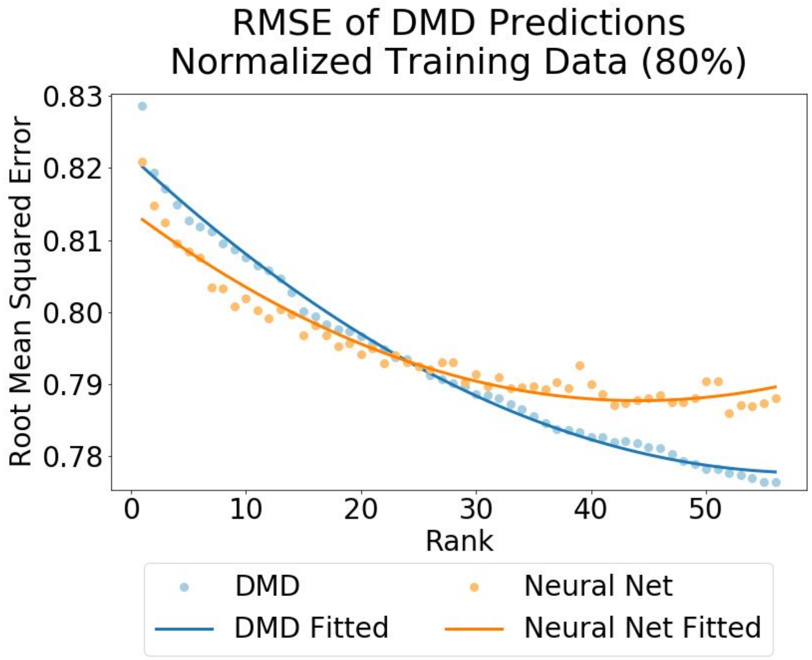
$$\tilde{A} = \phi \begin{pmatrix} \lambda_1 & & & & \\ & \lambda_2 & & & \\ & & \lambda_3 & & \\ & & & \lambda_4 & \\ & & & & \lambda_5 \end{pmatrix} \phi^{+}$$

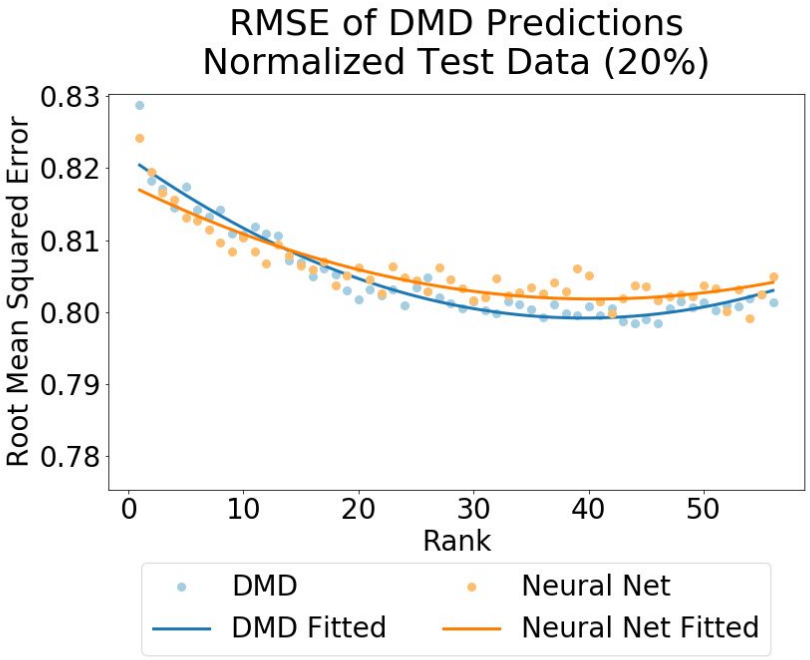
$$\begin{pmatrix} \\ Q_{t+1} \\ \\ \end{pmatrix} = \tilde{A} \begin{pmatrix} \\ Q_t \\ \\ \end{pmatrix}$$

Supervised Machine Learning



Prediction using DMD

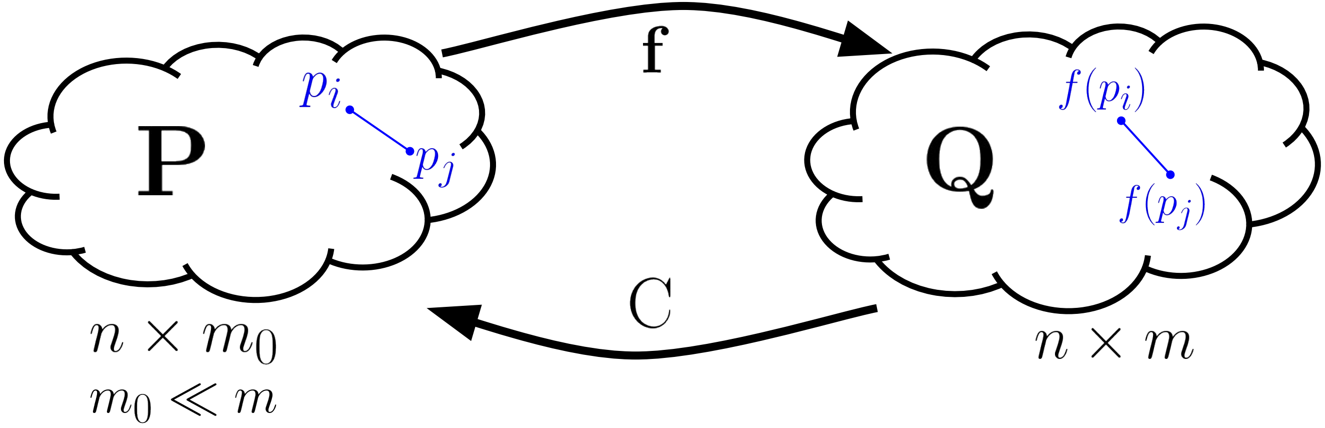




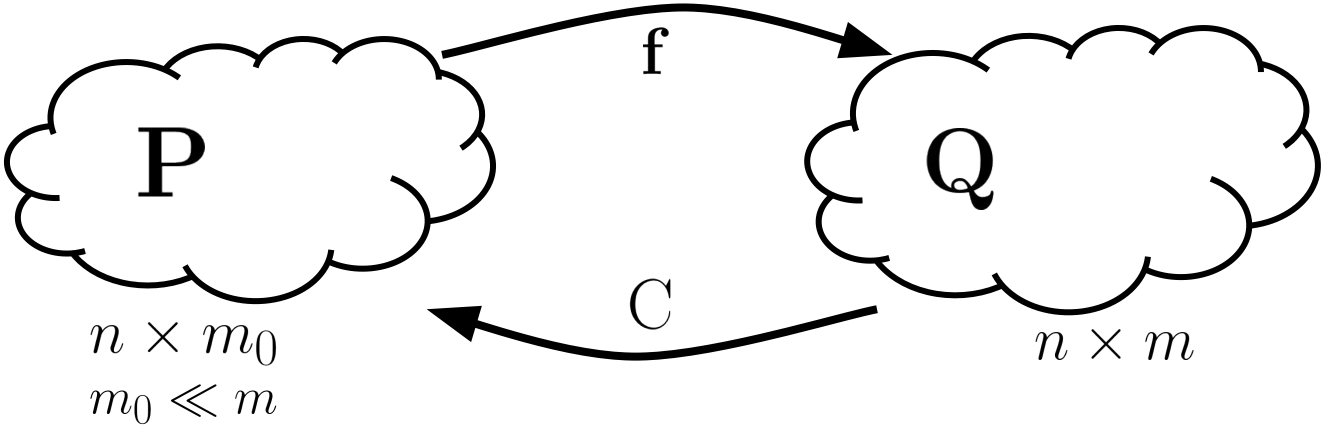
Dimensionality Reduction

Goal:

Given a matrix $\mathbf{Q} \in \mathbb{R}^{\mathbf{n \times m}}$, find $\mathbf{P} \in \mathbb{R}^{\mathbf{n \times m_0}}$ and $\mathbf{f} : \mathbb{R}^{\mathbf{m_0}} \to \mathbb{R}^{\mathbf{m}}$ such that $\mathbf{f}$ is approximately isometric.



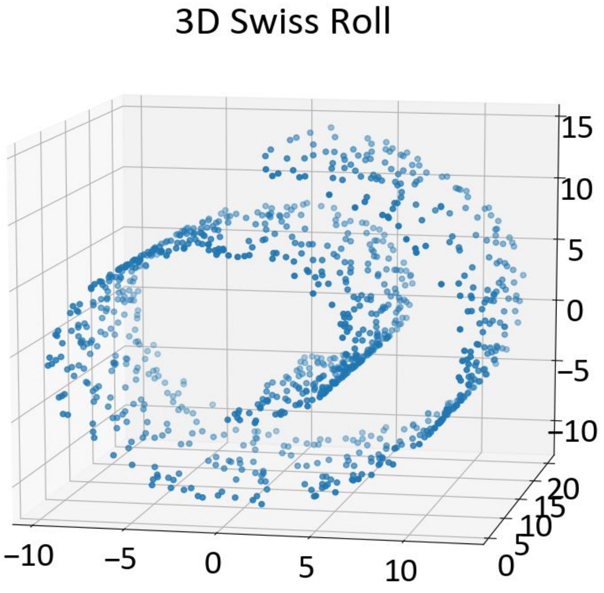
Isometry: $d(p_i, p_j) = d(f(p_i), f(p_j))$

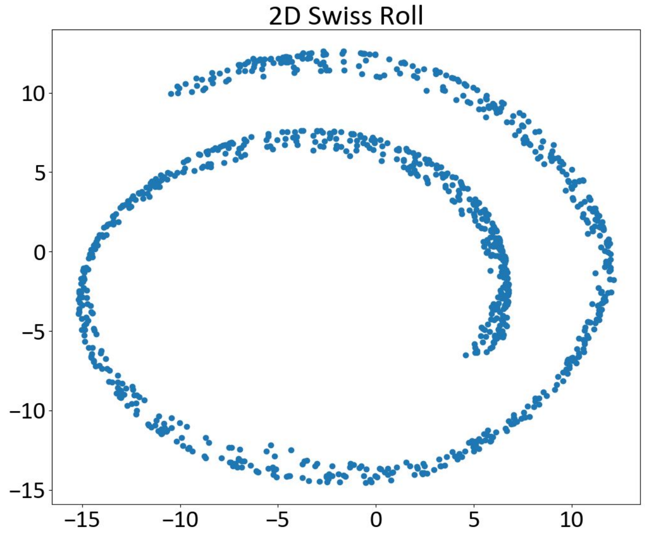


P
f
Q
C
$n \times m_0$
$m_0 \ll m$
$n \times m$

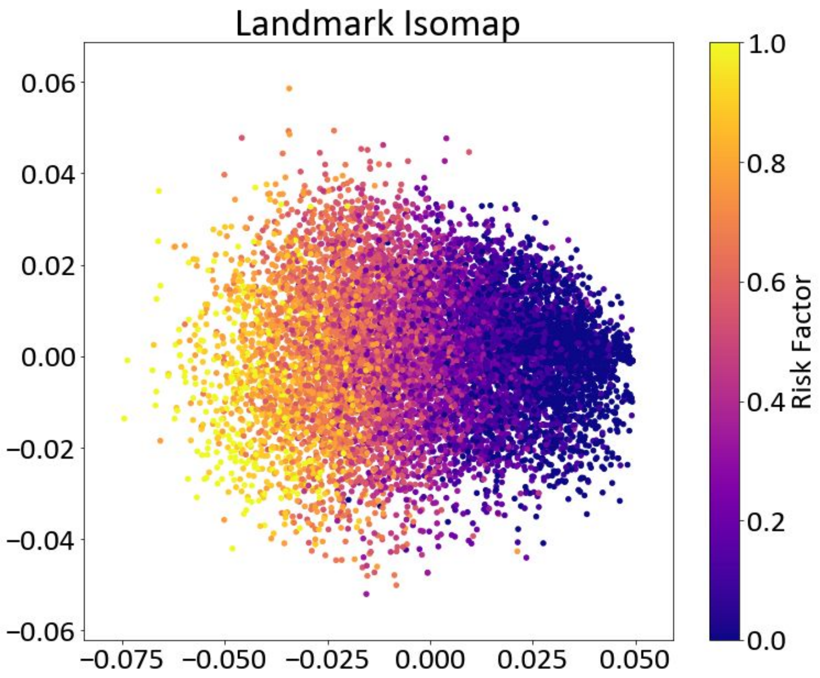
Landmark Isomap

1. Compute $\mathbf{P}_0$





Landmark Isomap



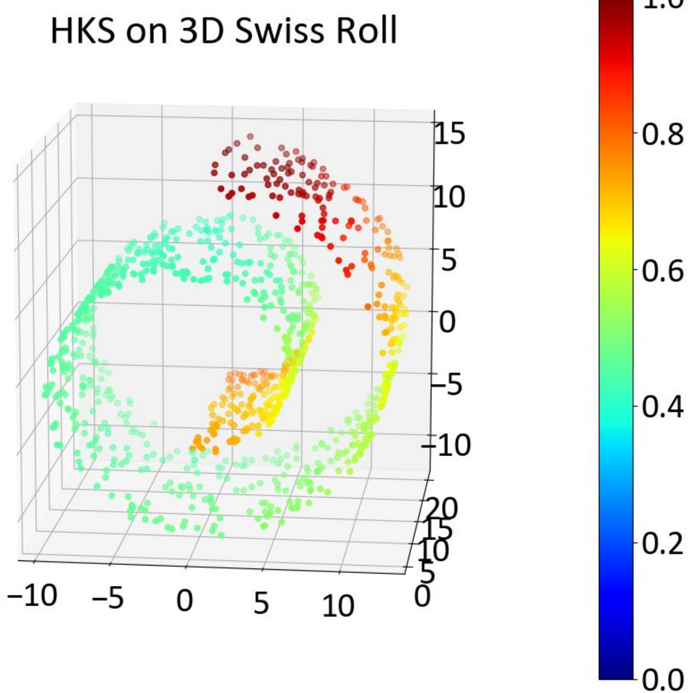
2. Construct Functional Constraints:

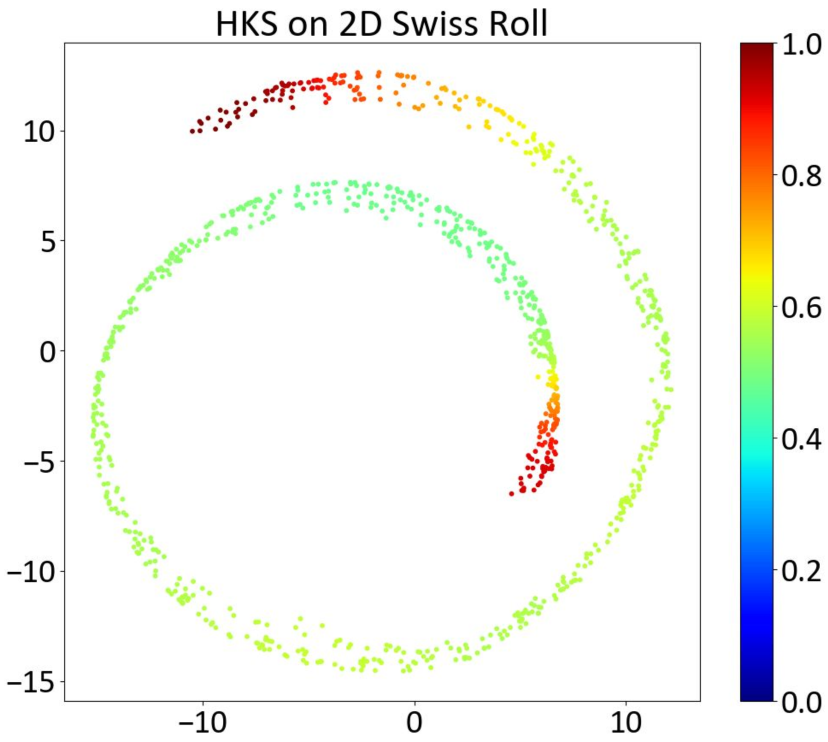
$$\mathrm{C}_k \underset{n \times r}{\mathbf{F}_{\mathrm{Q}}} = \underset{n \times r}{\mathbf{F}_{\mathrm{P}_k}}$$

Heat Kernel Signature (HKS)

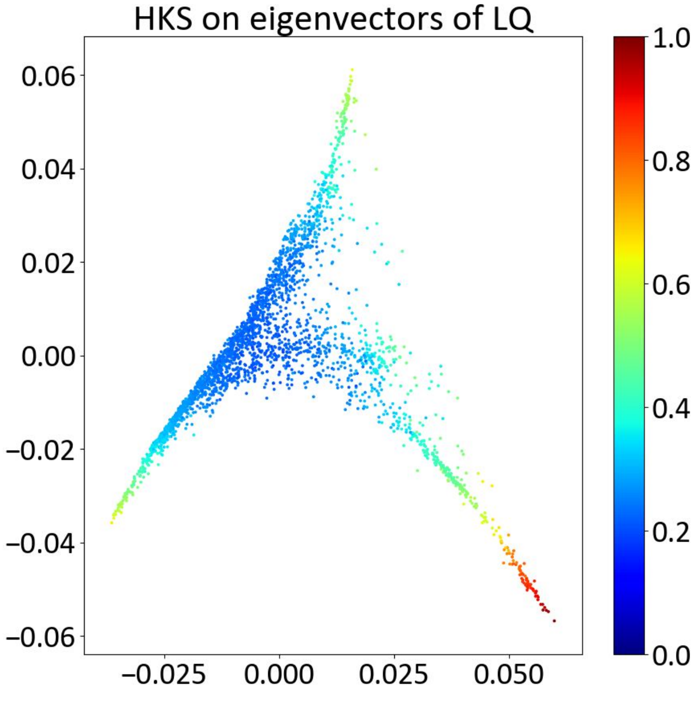
- Dissipation of heat from the point onto the rest of the shape over time.
- Short time: highly local shape features
- Long time: summaries of the shape in large neighborhoods
- Match between points by comparing their signatures at different time intervals

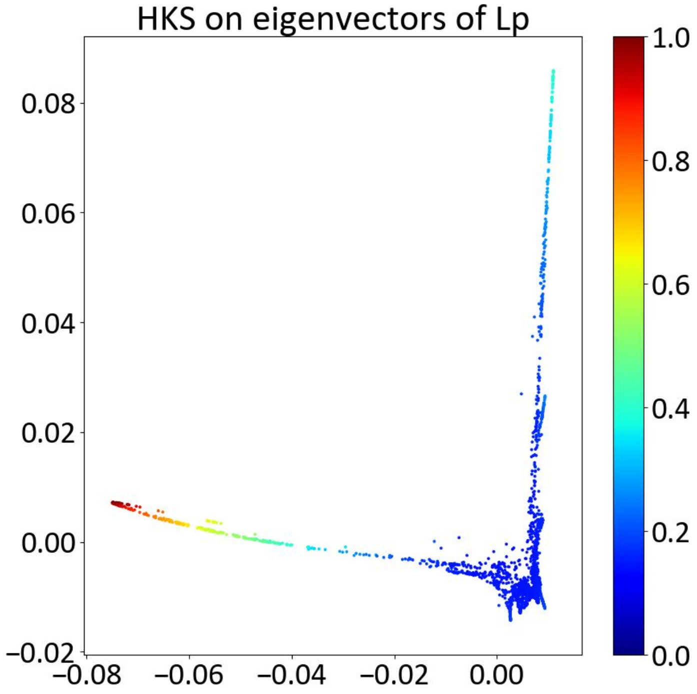
HKS on Swiss Roll





HKS on GRYD data





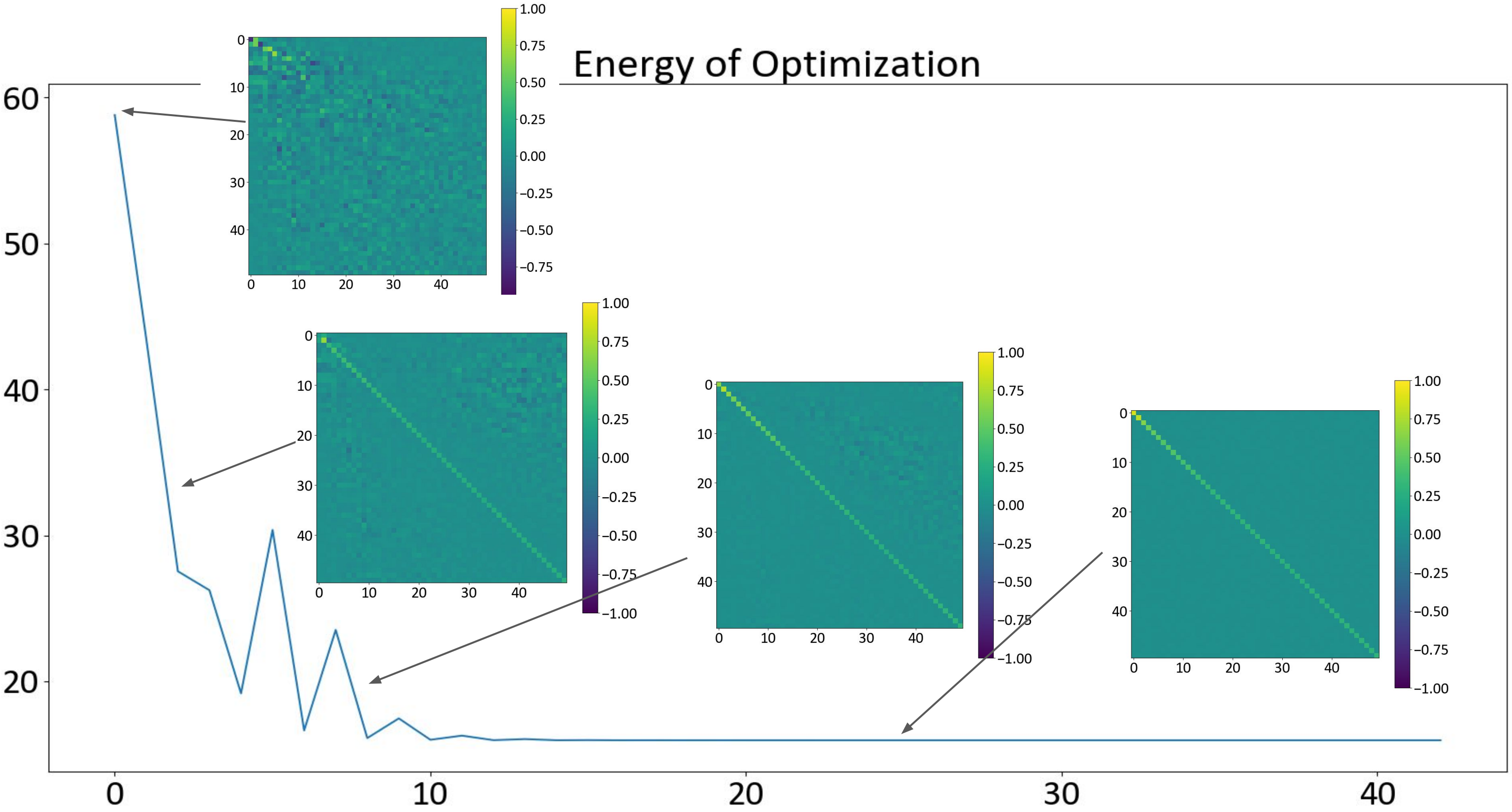
Optimization

3. Solve $\arg\min_{C_k} \frac{1}{2}\|C_k F_Q - F_{P_k}\|_F^2 + \frac{1}{2}\|C_k L_Q - L_{P_k} C_k\|_F^2$

Procedure:

- Perform SVD on F_Q and find U_Q .
- Transform F_Q, F_P, L_Q and L_P.
  - eg. $\hat{F_Q} = U_Q^T F_Q, \hat{L_P} = U_Q^T L_P U_Q$
- Run Optimization.
- Reconstruct C_k

Optimization



Find a New Embedding

4. Extract f^{-1} and $\mathbf{P}_{k+1}$ from C_k

- Apply delta function

5. Go back to step 2 and iterate

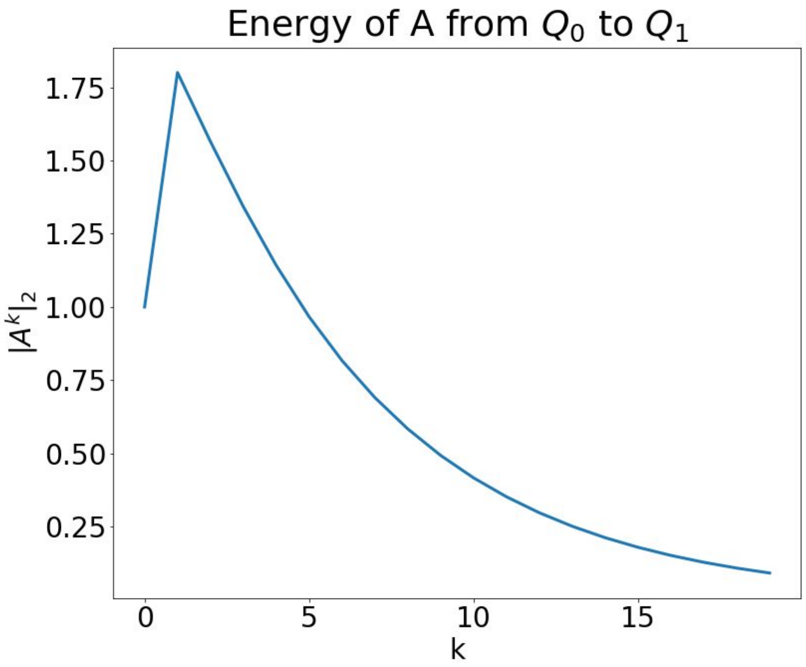
Problems

- Optimization sticks at local minimum
- HKS does not capture enough features.

Future Work

Why do we see the change that we see?

- Transient growth
- Resiliency of questions
- Dimension reduction



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