Experimental and Theoretical Investigation of Constant Flux Bidensity Particle Laden Flows on an Incline

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Introduction

Particle laden flow is the flow of a fluid with immiscible particles suspended in it. We study bidensity thin film particle laden flow on an incline with the constant flux initial condition. Previous research[1,3,4] has utilized experiments and mathematical techniques to develop a force-based model. For constant volume monodisperse flow, shear induced migration causes particles to move upwards (inducing the emergence of a 'ridged' regime), while gravitational settling causes them to move downwards (inducing the emergence of a 'settled' regime). Lee et. al.[2] and Wong and Bertozzi[5] extended this model to the bidensity case using experiments and theory, respectively. They used constant volume initial conditions, meaning that a fixed volume of slurry was poured onto the incline and allowed to flow. We adapt their model to use constant flux initial conditions, where the slurry is continuously added to the incline with a constant flow rate. We also perform experiments using the same total volume fraction Φ = 0.4 parameter, and compare the results to the prediction of the theory.

Set Up

a) $$\phi = \frac{V_{particles}}{V_{total}}$$

b) $$\chi = \frac{\phi_{Glass}}{\phi_{Glass} + \phi_{Ceramic}}$$

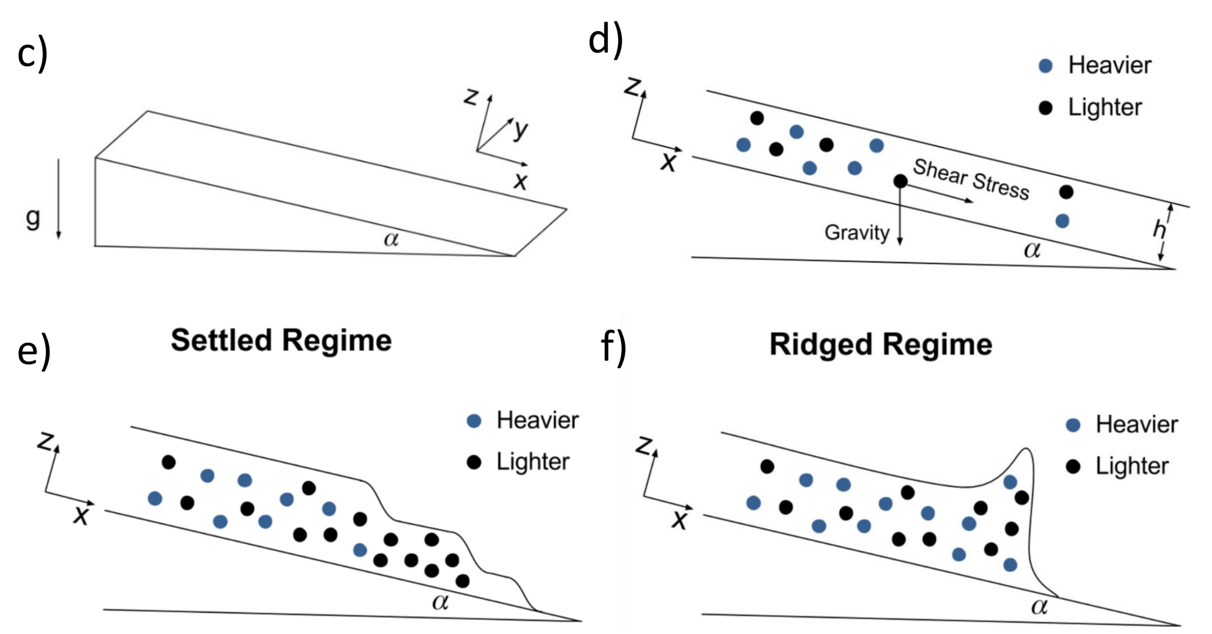

Figure 1 a) equation describing the total volume fraction *Φ* b) description of volume ratio of beads *χ* c) view of the track with coordinates d) side view of the track with forces e) typical settled regime behavior: heavier particles settle out of the flow, followed by lighter particles, leaving a clear fluid front. f) typical ridged regime behavior: particles migrate to the front of the flow, creating a ridge.

Particles	Density ρ	Diameter d	Color
Glass	2.5 g/cm³	0.04 – 0.06 cm	White
Ceramic	3.8 g/cm³	0.04 – 0.06 cm	Red
Fluid	Density ρ	Viscosity ν	Color
PDMS oil	0.971 g/cm³	100 cm²/s	Clear

Table 1 Materials used in all experimental trials. The ratio of particles in all slurries, determined by total volume fraction Φ, is 0.4.

Theory

Our goal is to model the height and the front position of the fluid and two particles. We will start with Navier-Stokes equation (1) and particle mass conservation (2). We define Φ_i, i = 1, 2 to be the volume fraction of particle i to the total volume, $\Phi = \Phi_1 + \Phi_2$, and the $\boldsymbol{J}_i$ term is the flux of particle i.

$$0 = \nabla \cdot \left(-pI + \mu(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)\right) + \rho \boldsymbol{g}. \qquad (1)$$

$$0 = \frac{\partial \phi_i}{\partial t} + \boldsymbol{u} \cdot \nabla \phi_i + \nabla \cdot \boldsymbol{J}_i, \quad i = 1,2 \qquad (2)$$

We also assume equilibrium in the *z*-direction. The result is a system of coupled ODEs relating the derivative of the shear stress *σ*, viscosity *μ*, and density *ρ*.

By introducing depth-averaged particle concentration and volume ratio , which can be calculated using *σ, μ*, and *ρ,* we derive a system of PDEs from conservation of fluid and particle mass,

$$0 = \frac{\partial h}{\partial t} + \frac{\partial}{\partial x}\left(h^3 f(\bar{\phi}, \bar{\chi})\right)$$

$$0 = \frac{\partial (h\bar{\phi}_i)}{\partial x} + \frac{\partial}{\partial x}\left(h^3 g_i(\bar{\phi}, \bar{\chi})\right), \quad i = 1,2$$

We consider the Riemann initial condition:

$$h(x,0) = \begin{cases} \left[\dfrac{3\mu(\phi_0)Q}{W\rho(\phi_0)g\,sin\alpha}\right]^{1/3}, & x < 0 \\ \varepsilon, & x > 0 \end{cases} \qquad \bar{\phi}(x,0) = \begin{cases} \phi_0, & x < 0 \\ 0. & x > 0 \end{cases} \qquad \bar{\chi}(x,0) = \begin{cases} \chi_0, & x < 0 \\ 0, & x > 0 \end{cases}$$

Expressing the system in vector notation we get

$$U = (h, h\overline{\phi_1}, h\overline{\phi_2})^T, \quad F(U) = h^3(f, g_1, g_2)^T \quad \Longrightarrow \quad U_t + \left(F(U)\right)_x = 0$$

We then solve the PDE using upwind scheme. And the height profile can be obtained from h. The particle front profile can also be found by looking at the concentration profile.

Numerical Results

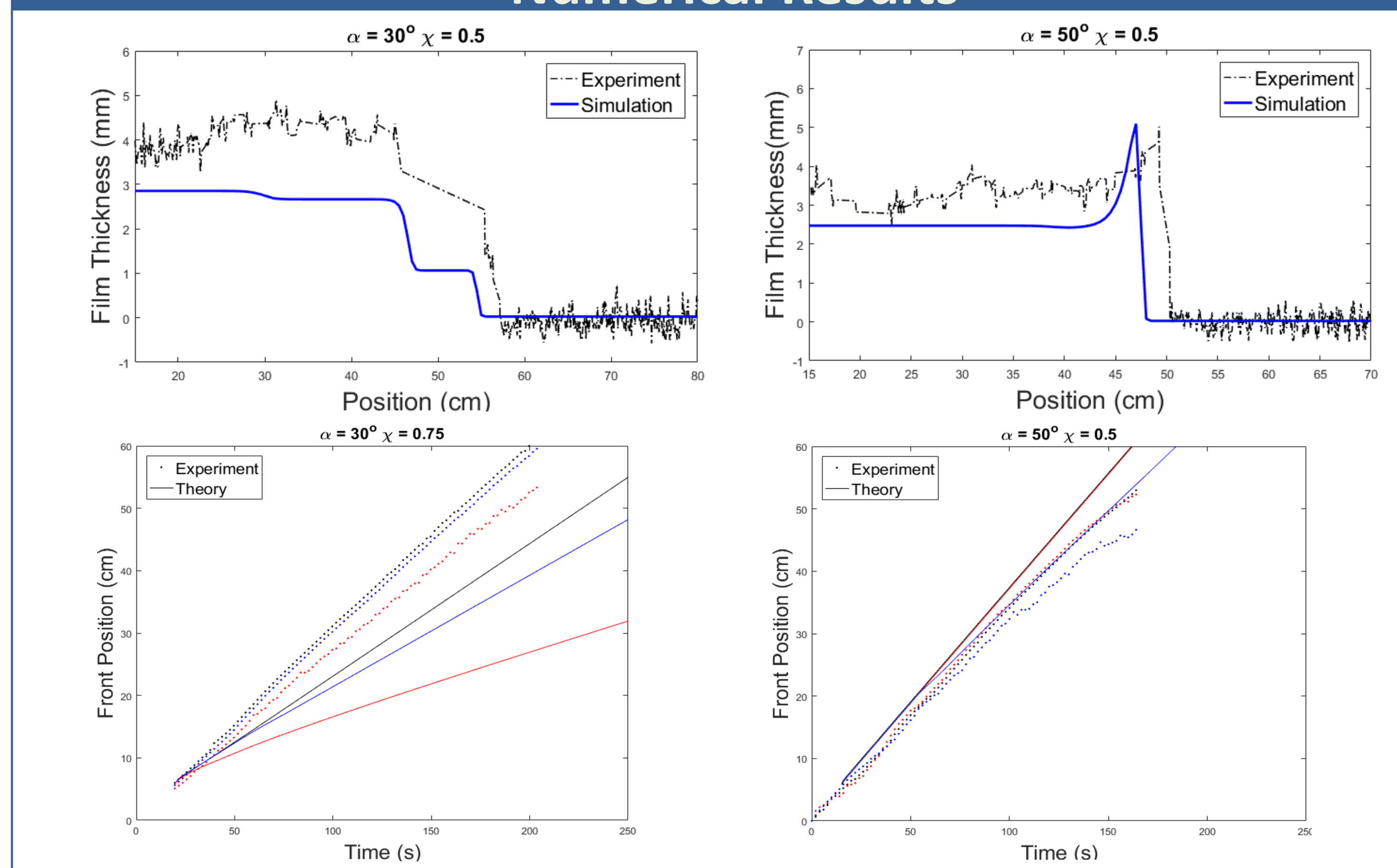

Figure 2 Experimental results superimposed with simulations generated by mathematical model. Clockwise from top left: height profile of settled regime, height profile of ridged regime, front position of ridged regime, and front position of settled regime.

Experimental Results

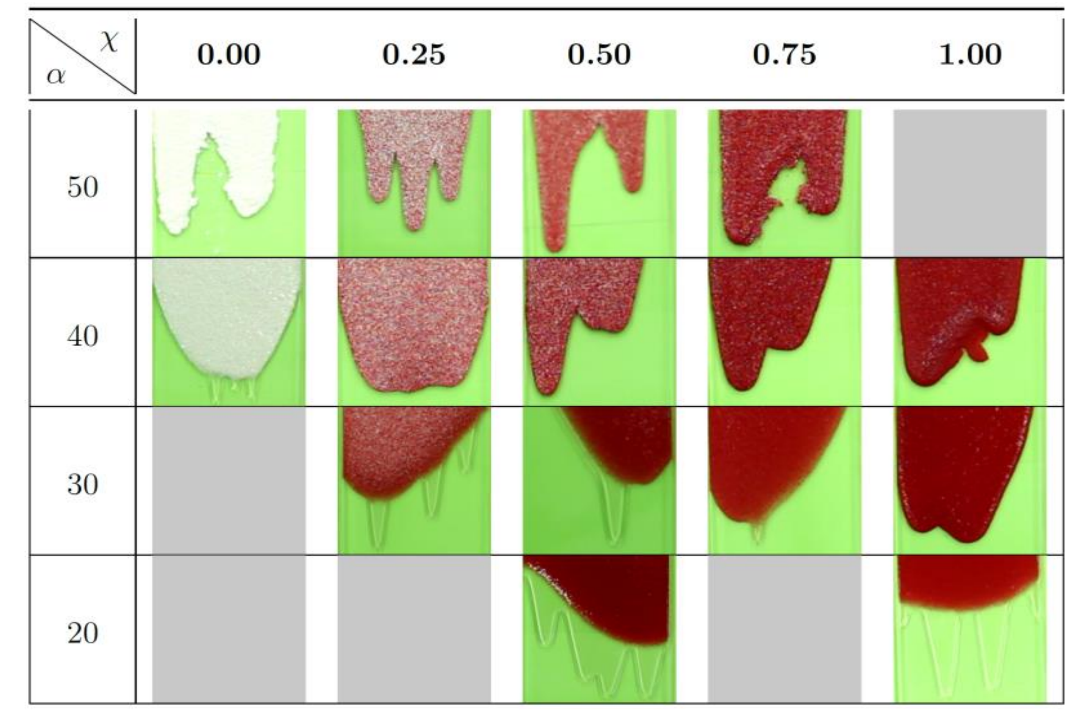

Figure 3: Experimental Results Small angles result in the settled regime, while large angles result in the ridged regime. Similarly, small particle concentration χ results in the settled regime, while high particle concentration results in the ridged regime.

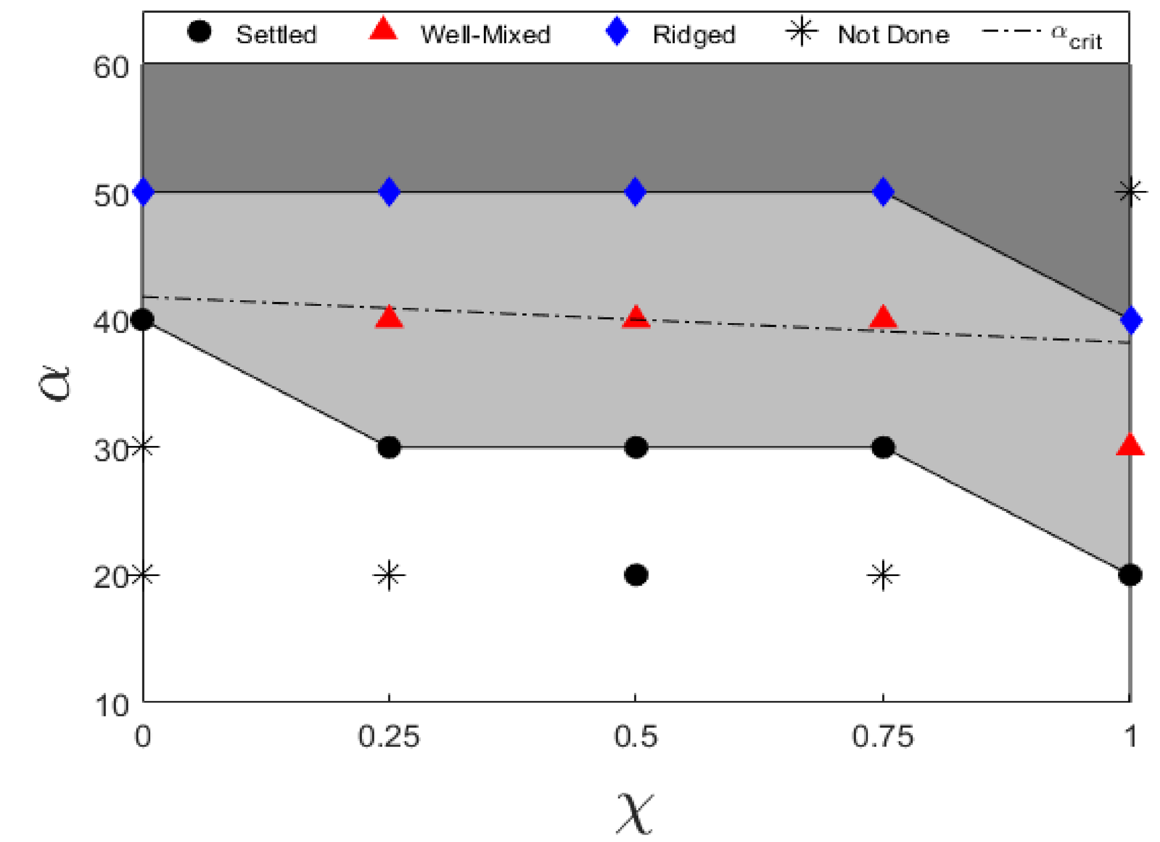

Figure 4: Phase Diagram Markers specify experimental trials. Shading indicates the ridged (dark grey), well-mixed (light grey), and settled (white) regimes. The theory suggests that eventually, the boundary between settled and ridged regimes is the dotted line.

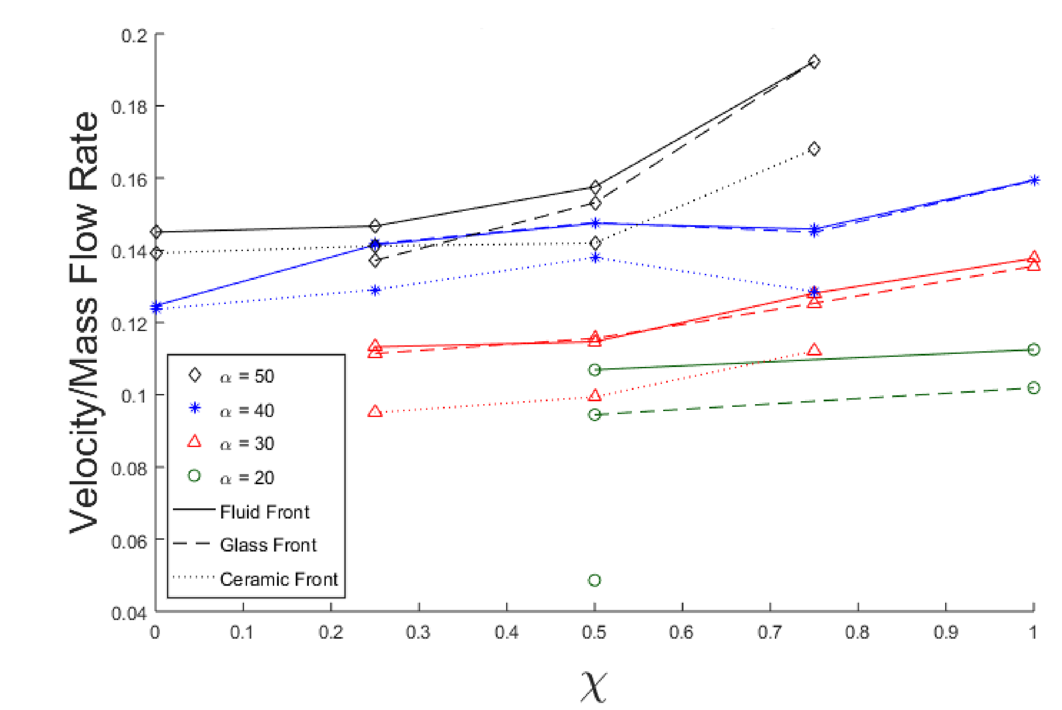

Figure 5: Velocity/Mass Diagram Velocities of all fronts for all settled trials, normalized over the rate of mass added per second.

Conclusion

We find that the behavior of the constant flux bidensity flow is similar to the constant volume case studied in previous work[2,5]. Experimentally, we determine the time required for a regime to emerge, and the critical angle at which the transition between settled and ridged regimes occurs. In addition, we implement a theoretical model that shows the same results. The comparison between the theory and experiment shows good agreement, especially in the ridged regime. Unlike in previous work, the front position grows linearly with time because of the constant flux initial condition.

Future Work

The model predicts the ridge to grow arbitrarily large in height over the course of the experiment. Further research could look into mitigating this behavior by including a precursor height, i.e. a thickness of the fluid that is already present on the incline before the front reaches it. It may also be useful to look into improving the experimental technique used to measure the height profile to reduce noise. Our model may also be modified to investigate other types of slurries, such as those with different size particles.

Literature Cited

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Acknowledgements

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